EXAM 1

Master Thm: $T(n) = aT(n/b) + f(n)$

① $f(n) = O(n^{\log_b a - \epsilon})$] $T(n) = \Theta(n^{\log_b(a)})$

② $f(n) = \Theta(n^{\log_b(a)} \log^k(n))$

$T(n) = \cancel{\Theta(f(n)\lg(n))}$ $\Theta(n^{\log_b a} \log^{k+1}(n))$

③ $f(n) = \Omega(n^{\log_b a + \epsilon})$] $T(n) = \Theta(f(n))$

$T(n) = 4T(n^{1/4}) + \lg^2 n$

Let $n = 2^m$

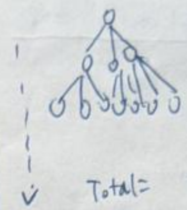
$T(2^m) = 4T(2^{m/4}) + m^2$

$S(m) = T(2^m)$

$S(m) = 4S(m/4) + m^2$

$T(2^m) = S(m) = \Theta(m^2)$

$T(n) = \Theta(\lg^2 n)$



$\frac{5}{6}n$

$(\frac{5}{6})^2 n$

$h = \lg_3 n$

Total $= \sum (\frac{5}{6})^i n$ + leaves $= \Theta(n)$

MSTs $G = (V, E)$, (u,v) light, must be in MST.

$\boxed{\text{Kruskals}}$ – $O(E \lg V)$

$T \neq \phi$
Make_set(v) $\forall v$ —— $O(|V|) \cdot T_{make\_set}$
Sort E by w —— $E \lg E$
for edge in E: —— $O(E)(T_{find\_set} + T_{union})$
  if find_set(u) ≠ find_set(v):
    add e to T.
    UNION(u,v)

$T = O(E \lg E) + O(V) T_{make\_set} + O(E)(T_{find\_set} + T_{union})$
$E \log V$

$\boxed{\text{Prims}}$: $O(V) \cdot T_{extract\_min} + O(E)\, T_{decrease\_key}$

$T = \phi$
Q: u.key = ∞ ∀u∈V, u.parent = None | $O(V)$
r.key = 0
while Q ≠ ∅:
  u = extract_min(Q) ⟵ extract min
  T = T ∪ {u, u.parent}
  for each v∈G.Adj[u]: ⟵ E things
    if v∈Q & w(u,v) < v.key:
      v.parent = u
      v.key = w(u,v) ⟵ Dec key

Using fib heap: $T = O(E + V \log V)$

If exist restrictions on edge weights, extract_min, decrease key could be $O(1)$.

$\boxed{\text{FFT}}$: samples ⟷ coeffs $O(n \lg n)$

$A(x) \cdot B(x) = C(x)$

$A^* = DFT(A)$] $n \lg n$
$B^* = DFT(B)$
$\vec{C}^* = \vec{A}^* \cdot \vec{B}^*$ — n
$C = IDFT(C^*)$ — $n \lg n$

→ $A(x) = A_{even}(x^2) + x A_{odd}(x^2)$

$|x|$ collapses by roots of unity

$T(n, |x|) = 2 \cdot T(\frac{n}{2}, |x^2|) + O(n)$

$|x^2| = \frac{|x|}{2}$

Minkowski:
2 sets X, Y → compute all possible sums

$P_X(x) \Rightarrow$ coeff 1 if x^k k∈X

P_{XY} FFT

#nonzero coeffs.

X, Y bounded by m → $O(m \lg m)$ or $O(n)$ — add m to each elem?

* Polynomial ~~cannot~~ have negatives.
* p much only on a BOUNDED range

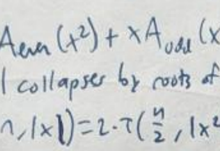
Div n Conquer: Divide into n/b, solve recursively, Combine solutions of subproblems to get solution.

$T(n) = a \cdot T(\frac{n}{b})$ + combine time

Median/rank find: $O(n)$

L	X	R

Choose X. Recurse if necessary on L/R.
$T(\frac{n}{x}) + O(\text{choose})$

1. Divide n elem into n/5] $\frac{n}{5} \cdot O(1)$
2. Sort, find median
3. Find median of n/5 group medians recursively $T(\frac{n}{5})$
4. 

med > median $= 1 \cdot \frac{n}{5} \cdot \frac{1}{2}$
all must > median
$2 \cdot \frac{n}{5} \cdot \frac{1}{2}$ → $\frac{3}{10}$ total.
n/5
7/10 max size group recursion

$T(n) = T(\frac{3}{4}n) + T(\frac{n}{5}) + O(1)$
$T(n) < cn$ by induction

→ suppose true for < m.

$T(n) \leq T(\frac{n}{5}) + T(\frac{3n}{4}) + c_2 n$
$\leq \frac{c_1 n}{5} + \frac{3c_1 n}{4} + c_2 n$
$\leq \frac{19}{20} c_1 n + c_2 n$
$\leq c_1 n + (c_2 - \frac{c_1}{20}) n$
$< c_1 n$ when $c_2 > 20 c_2$.

$\boxed{\text{Networks}}$

$|f|$ = flow going out of s. $|f| = f(S)$ – flow across any cut S ∀S

$F^* = \max$ flow $= |f^*|$

$c(S)$ = capacity of cut S (caps out) $f(S) \leq c(S)$

$f(S)$ = flow across cut S

Capacity of a path $= \min_{e \in p} c_f(e)$

Max Flow Min Cut Thm $F^* = c(S^*)$ → mincut (cut w/ min Σ caps)
↳ f admits no augmenting path
↳ no s→t path in G_f (where the min cut happens)

Edmond-Karp = $O(E^2 V)$

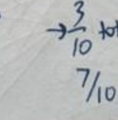
* DFS, BFS to find an edge or some path s→t run in $O(|E|)$
* Dealigment? Think residual graph, DFS.
  DFS u→s, $c_f(u,v) -= 1$; $c_f(v,u) += 1$
  ∃ new s→t path in G_f? DFS

$(1 - \frac{1}{k})^k \leq e^{-1}$

* Find a mincut? Use BFS in G_f to find ∀u∈S, some edge e(u, v∉S)
  – Min cut might not be unique
  – Max flow network also not unique

$\boxed{\text{UNION FIND}}$

Make_set $O(1)$
Find_set $O(1)$] m ops $= O(n \lg n + m)$
Union – all unions → $n \lg n$

Forest: 

Union(u,v) = $O(h_u) + O(h_v)$
Find $= O(h)$

Final: m ops → $m\alpha(V)$

Path compression every find_set connects vertices head to root

Max Flow EX

~~e~~ edge e → cap -= 1
  Max flow changes?
If c ≠ f(e), do nothing
else: DFS u→s, t→v, decrement resid capacity in G_f, increment back edge
New augmenting path? Yes → no change
No → $|f| -= 1$

Competitive Analysis / Amortized

MTF: after accessing x, move x to front

Cost = $rank_L(x) + rank_L(x) - 1 = 2rank_L(x) - 1$

$C_A(s) \le \alpha \cdot C_{OPT}(s) + k$

Φ = (# invs b/w L_i & L_i^*) $\cdot 2$

$\Phi(0) = 0$

$\Delta\Phi = \pm 2$

L_{i-1}: $\boxed{A \cup B \mid x \mid C \cup D}$ $r = |A| + |B| + 1$

L_{i-1}^*: $\boxed{A \cup C \mid x \mid B \cup D}$ $r^* = |A| + |C| + 1$

MTF step → create $|A|$ inv, destroy $|B|$ inv

OPT → each t_i creates ≤ 1 inv

$\Delta\Phi \le 2(|A| - |B| + t_i)$

$\hat{c}_i \le \underbrace{2r-1}_{c} + 2(|A| - (r-1-|A|) + t_i)$

$\le 4(r^* + t_i) \le 4c_i^*$

MTF is 4-competitive

AGGREGATE

$\hat{c} = \frac{\text{Total cost of } k \text{ ops}}{k}$

Accounting:

Table Doubling:

$n/2$ elem → $n/2$ coins

- Use 1/2 coins for n doubling
- Total $\to O(1) \cdot \frac{cn}{2} = 0$
- Amortized/insertion → $1 + c = O(1)$

MST Ex: Bottleneck spanning tree

$v(T)$ = max edge

Given G, b, determine whether minimal $v(T) \le b$.

- Remove all $e \in E$, $w(e) > b$.
- Run DFS.

In General, think DFS, BFS, changes to Prims, Kruskals

Potential Method:

Tree: Φ - $\lg n$

Stack: # elems in stack

$\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$

$\sum_i^n \hat{c}_i = \sum_i^n c_i + \Phi(D_n) - \Phi(D_0)$

Ex

PUSH → create elem, place on top. $O(1)$

POP → return/remove top elem $O(1)$

FUTUREPOP → Reads top key, removes k elems for $O(k)$, returns value of kth elem after that $O(k)$

Let Φ = 2 · # elem in data-struct

Push $\hat{c} = c + \Delta\Phi = 1 + 2 = 3$

Pop $\hat{c} = c + \Delta\Phi = 1 - 2 = -1$

FUTUREPOP $\hat{c} = c + \Delta\Phi = 2k + 1 - 2k = 1$

Find an edge on the min cut → for a given max flow

Run BFS to detect S of edges reachable from s. Iterate thru all edges to find an edge in S going out of S, return.

FFT Ex

$P = \{-1, -4, -5, 2, 3\}$

Triples (x, y, z) $\Sigma = 0$

Bound $(-314, 314)$

Add d to every elem.

- P^3 by FFT - coeffs that precedes x^{3d} = # things $\Sigma 0$.
- Paired sums → original polynomial, $\forall i \in P$, w/ add d.

$c\,x^{2i}$ $c = 1$.

- multiply by FFT P.
- Subtract $3c_{3d}$ from P^3
- If coeff > 0, return T.

P_+ = [pos elem] P_- = [neg elem]

$A_+(x) = \sum_{i \in P_+} x^i$ $B(x) = A_+^2(x) - A_+(x^2)$ $n \lg n$

$\parallel$

2 distinct numbers sum to i

For all nonzero coeffs, $-i \in P_-$, then yay

If you take an element out, it's literally

$(P_A - x^a)(P_B - x^b) = P_A P_B - x^a P_B - x^b P_A + x^a x^b$

Still no added time

You need at least $d+1$ samples

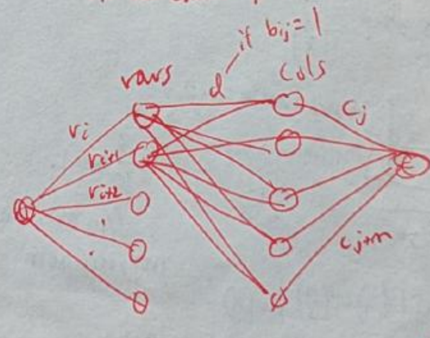
Flow Ex

Grid n, m each row has at most r_i flowers

each col has at most c_j flowers

only flowers can be placed in cells $b_{i,j} = 1$

All can contain up to d flowers.



Edges = $nm + n + m = O(nm)$

FF → $O(|E||f|) = O(nm(nmd))$

Finding residual edge w/ min max capacity : $m \lg n$ path w/

Guess u, max weight of min edge from $s \to t$

Sort m edges $m \lg n$

Binary search, optimal u in $\lg n$ DFS

Ford Fulkerson: start w/ 0 flow

Augmenting path via DFS

Augment flow by pushing $c_f(p)$ along path p

Repeat until none left

$O(E \cdot V \cdot C) = O(|E||f|)$

Binary Counter: Increment: Add 1 → worst case all 1s must switch to 0s

Let Φ = # 1s

Cost = $k + 1$ $\Phi = -k$

$\hat{c}_i = 1$

EXAM #2

LP

min → max $\vec{c} \to -\vec{c}$

$\geq \to \leq$ mult both sides by -1

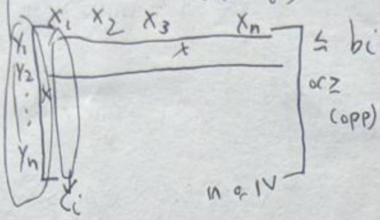
$= \to$ use both $\leq$ & $\geq$, $\neq 2$

$x \in \mathbb{R} \to x \geq 0$, $x_i^+ \geq 0$, $x_i^- \geq 0$

$x \to (x_i^+ - x_i^-)$

max Standard form: max $\vec{c}\vec{x}$ ST $A\vec{x} \leq \vec{b}$, $\vec{x} \geq 0$

min $\vec{b} \cdot \vec{y}$, $\vec{A} \cdot \vec{y} \geq \vec{c}$, $\vec{y} \geq 0$



• max a minimum? set min = q, ST $\forall x \geq q$.

Random Walk

p_v^t - prob at being v at time t

$$p_v^{t+1} = \sum_{e=(u,v)} \frac{1}{d(u)} p_u^t$$

$p^{t+1} = Wp^t = W^{t+1} p_0$ nonlazy

$\hat{W} = p_{lazy} I + (1-p_{lazy}) W$

$\hat{p}^{t+1} = \hat{W}\hat{p}^t = \hat{W}^{t+1} \hat{p}_0$ lazy

Lazy have self loops $\forall v$

$$p_v^{t+1} = p_{lazy} p_v^t + (1-p_{lazy}) \sum_{e=(u,v)} \frac{1}{d(u)} p_u^t$$

$A = \begin{bmatrix} 1 & (v,u) \in E \\ 0 & ow \end{bmatrix}$ $D = \begin{bmatrix} d(u) & \text{if } u=v \\ 0 & ow \end{bmatrix}$

$W = AD^{-1} = \begin{cases} 1/d(v) & \text{if } (v,u) \in E \\ 0 & ow \end{cases}$

Does it converge to stationary dist? What is it?

Write out $\pi_0 \to \pi_1$, $\pi_1 \to \pi_2$, $\pi_2 \to \pi_3$, π_n in terms of π_0

Stationary π: $W\pi = \pi$

$$\pi_v = \frac{d(v)}{\sum_{u \in V} d(u)}$$

Which has a stationary dist to which walks converge?

Every connected, nonbipartite undirected graph

→ Lazy walks → connected, undirected

→ Directed - strongly connected, aperiodic, $\exists$ self loops

Hashing

n = # keys in table

m = # slots

SUHA - $\Pr_{k \neq k'}[h(k) = h(k')] = \frac{1}{m}$, $\Theta(1+\alpha)$ lop, $\alpha = n/m$

Universal Hashing: $\mathcal{H}$ is uni if $\Pr_{h \in \mathcal{H}}[h(k) = h(k')] \leq \frac{1}{m}$,

k, k': $E[\#\text{keys in a slot}] \leq 1+\alpha$, $\alpha = \frac{n}{m}$

To prove something isn't uni, take a (k, k') and prove w/ random h's, it collides a lot.

Perfect Hashing:

but l_j^2 slots; l_j := # elems in slot j

$\to$ if $\sum_{j=0}^{m-1} l_j^2 > cn$, hash again

$E\left[\sum l_j^2\right] = O(n)$, $m = \Theta(n)$

$\Pr[\text{collision}] \leq 1/2$ by Markov

2-universal

For some $x_1, x_2 \in U$, $x_1 \neq x_2$: h is randomly selected from $\mathcal{H}$,

$(h(x_1), h(x_2))$ is equally likely to be one of m^2 pairs (y_1, y_2)

Pairwise Ind = universal!

$$\Pr_{\substack{h \in \mathcal{H} \\ x_1 \neq x_2}}[h(x_1) = y_1 \wedge h(x_2) = y_2] = \frac{1}{|Y|^2}$$

Game Theory

Utility matrix A

	B_1	B_2	B_3	...
A_1	Reward A gets			
A_2				
⋮				
A_n				

$\exists$ a stable outcome w/ pure determ. strategies?
- Take some examples, see if oscillation.

$\exists$ stable outcome from random assign?
- Are all E[utility] equal for all possible actions
- Is there incentive to switch after assignment?

Utility B is exact same, but w/ reward from B

Stable outcome → Nash Equil.

- For 2-person 0 sum games, general games w/ finite # players, actions

V_R := expected utility of row player if they go first

V_C := expected negative utility of col player if they go first

min Max Thm: $V_R = V_C = V$

$$V_R = \max_{x \in P} \min_{y \in Q} xAy \to x^*$$

$$V_C := \min_{y \in Q} \max_{x \in P} xAy \to y^*$$

LP: $V_R = \max z$

$(\sum_j A_{ij} - x_i) - z \geq 0$

$\sum x_i = 1$

$x \geq 0$

Streaming - limited space

Reservoir sampling - $\forall i$, keep each elem i w/ prob $\frac{i}{i+1}$ / new elem $1/i+1$

Sample k random - Include first k, at each x_{i+1}, $\frac{k}{i+1}$, remove random elem from the k in reservoir

Frequency Moments: F_0: # distinct elems, d distinct, n elem total

```
z = 0
for each j in list:
    if zeroes(h(j)) > z:
        z = zeroes(h(j))
return 2^z
```

* For a Markov Chain, if W is doubly stochastic, uniform dist is stationary.

* PF Spanning tree - contains all vertices
  - minimal edges
  - connectedness

→ Markov Chain →
1. Strongly connected aperiodic? $\exists$ eq dist.
2. If $\pi = \pi W$, then π is stationary.
3. If eq exists, it is also unique stationary dist.

Randomized Alg

- Monte Carlo - runs fast, prob correct
- Las Vegas - prob runs fast, correct

Ex Matrix Product:

$A \cdot B = C$

$A \cdot (\underbrace{B \cdot r}_{O(n^2)}) = C \cdot r$

Correct? YES

Incorrect? $D = C - AB$, some D is nonzero. $\exists Dr \neq 0$ for some r.

$r_{Bad} + V = r_{Good}$, so $\exists$ 1 good for 1 bad. $\boxed{1/2 \text{ prob}}$ $v[j] = 1$

Expectation → Chernoff Bounds for bounding prob

Linearity of Exp → $E[X] = E\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n E[X_i]$

Union-Bound → $\Pr\left[\bigcup_{i=1}^n A_i\right] \leq \sum_{i=1}^n \Pr[A_i]$

Markov's Inequality → $\Pr[Y \geq a] \leq \frac{E[X]}{a}$, $a > 0$, $\forall Y > 0$

Chebyshev's $\Pr[|X - \mu| \geq k] \leq \frac{Var[X]}{k^2}$, $k > 0$

Chernoff: $X_1 \dots X_n$ ind, random $\{0, 1\}$

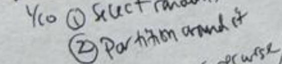
$\Pr[X > (1+\beta)\mu] < e^{-\beta^2 \mu/3}$ $0 < \beta < 1$

$\Pr[X > (1+\beta)\mu] < e^{-\beta\mu/3}$, $\beta > 1$

$\Pr[X < (1-\beta)\mu] < e^{-\beta^2 \mu/2}$ $0 < \beta < 1$

X_i mutually independent

Randomized Select - Find elem of rank k:



1. Select random x
2. Partition around it
3. return or recurse

- Reduce 9/10 every time:

$$T = \sum_{r=0}^{\log n} \Theta\left(\left(\tfrac{9}{10}\right)^r n\right)$$

- Don't hit 9/10 100% of time ... T_r = time to get good hit:

$$T = \sum \Theta\left(\left(\tfrac{9}{10}\right)^r n\right) T_r$$

$T_r \geq 5$, $\Pr[\text{not hit}] \leq 0.2^5$

$Ex[T_r] \leq 5/16$

$E[T] = O(n)$

Ex probability

$\Delta G = \#\Delta s \text{ in } E^3$

$X_i = \binom{n}{3}$ if $\{u,v,w\}$ create Δ

$X = \sum_{i=1}^{T} X_i$ $\quad E[X_i] = Pr[X_i = \binom{n}{3}] \cdot \binom{n}{3}$

$E[X] = \frac{\Delta G}{\binom{n}{3}} \cdot \binom{n}{3}$

$E[X] = E[\sum X_i] \cdot \frac{1}{T}$ $\quad E[X_i] = \Delta G$

a) $= \frac{1}{T} \sum_{1}^{T} \Delta G = \Delta G = E[X]$

$Var[X] = \frac{1}{T^2} \sum Var[X_i]$

$Var[X_i] = E[X_i^2] - [E[X_i]]^2$

$E[X_i^2] = \binom{n}{3}^2 \cdot pr = \binom{n}{3}^2 \cdot \frac{\Delta G}{\binom{n}{3}}$

$= \binom{n}{3} \Delta G$

$Var[X_i] = \binom{n}{3} \Delta G - \Delta G^2$

$Var[X] = \frac{1}{T^2} \sum^{T} \binom{n}{3} \Delta G - \Delta G^2$

$= \frac{\Delta G}{T} \cdot \left(\binom{n}{3} - \Delta G\right)$

Ex prob (...)

$X_i = 1$ if elems in range $\quad$ elems in range, $\Sigma X < \frac{3k}{2}$, w/ probability $1-\delta$

$X = \sum^{k} X_i$

$Pr[X < \frac{k}{2}] < \delta$ — Chernoff

$(1-\beta)\mu \quad e^{-\beta^2 \mu / 2}$

- $\mu = \sum^{k} E[X_i] = \sum Pr[X_i = 1] \cdot 1 = 0.6k$
- $\frac{k}{2} < (1-\beta) \cdot 0.6k$

$\beta < 1/6$

- Set β as $1/8$ or something

$\delta = e^{-\beta^2 \mu / 2}$

$-\ln(\delta) = \frac{0.6k\beta^2}{2}$

$k = O\left(\ln\left(\frac{1}{\delta}\right)\right)$

Ex Hashing

2-universal (not 3-uni)

	a	b	c
h_1	0	0	0
h_2	1	0	1
h_3	0	1	1
h_4	1	1	0

strongly 2-universal → universal

2-universal: $Pr_{h \in H}[h(x_1) = h(x_2)] = (i,j) \cdot = \frac{1}{m^2}$

for some i

$Pr[h(x_1) = h(x_2)] = \sum_{i=1}^{m} Pr[(h(x_1) = h(x_2)) = (i,i)]$

$= \frac{1}{m}$ → universal

Consider nash eq $\hat{x}A\hat{y}$ and $\tilde{x}A\tilde{y}$. Prove that they are equal.

$\hat{x}^T A \hat{y} = \max_x x^T A \hat{y}$ $\quad$ if row player chooses second when col player chooses $\hat{y}$

$\hat{x}^T A \hat{y} = \min_y \hat{x}^T A y$

$\hat{x}^T A \hat{y} = \max_x x^T A \hat{y} \geq \max_x \min_y x^T A y = V_R$

$\hat{x}^T A \hat{y} = \min_y \hat{x}^T A y \leq \min_y \max_x x^T A y = V_c$

$V_c \geq \hat{x}^T A \hat{y} \geq V_R$

So $V_c = V_R = \hat{x}^T A \hat{y}$.

Same arg for $\tilde{x}A\tilde{y}$.

Ex Proving Dot-Product Hash Fam is uni:

$h_a(k) = a \cdot k \bmod m$

$Pr_a[h_a(k) = h_a(k')] = Pr[\sum_i a_i k_i = \sum_i a_i k_i' \bmod m]$

$= Pr[\sum a_i(k_i - k_i') + a_d(k_d - k_d') = 0 \bmod m]$

$= Pr[a_d = -(k_d - k_d')^{-1} \sum_{i \neq d} a_i(k_i - k_i') \bmod m]$

$= E[Pr[a_d = f(k, k', a_d)]]$

$= \sum_x Pr[a_d = x] Pr[a_d = f(k, k', x)]$ $\quad$ f in m

$= \frac{1}{m}$

Ex LP

- n pet fish (i)
- m fish foods (j)

LP primal: q = min happiness of fish $\forall i$

- Food j cost c_j/unit
- Happiness = quantity (j) $\cdot H_{ij}$ of fish i
- d dollars

f_j = quantity of food j

→ max q

ST $q - \sum_{j=1}^{m} H_{ij} f_j \leq 0, \forall i: 1:n$

$\sum_{j=1}^{m} c_j f_j \leq d$

$f_j \geq 0 \; \forall j = 1:m$

$q \geq 0$

$$\begin{array}{cc} & \text{food } j \\ \text{fish } i \; y_i & \begin{bmatrix} 1 & -H_{i1} & -H_{i2} & \cdots & H_{im} \\ 1 & & & & \\ 1 & & & & \\ 1 & & & & \\ 0 & c_1 & c_2 & \cdots & c_m \end{bmatrix} \\ z \rightarrow & \end{array} \begin{bmatrix} q \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d \end{bmatrix} \forall i: 1:n$$

$1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0$

→ max $\langle q, \vec{f} \rangle \cdot \langle 1, \vec{0} \rangle$

Dual:

$\sum y_i \geq 1 \quad y_i, z \geq 0$

$c_j z - \sum_{i=1}^{n} H_{ij} \cdot y_i \geq 0$

max $d \cdot z$

Ex Streaming - Majority elem

Stream n ints → frequent ints appear > m = n/k times, some k

→ Initialize

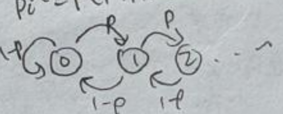
→ For i in stream:
- if i in map: map[i] += 1
- else: if empty slot in map: insert i
- else: $\forall$ slots in map: map[slot] -= 1; if 3 empty slot: insert i

return slots in map

- There could be < k-1 such frequent ints, so you might need to check the remaining k-1 elems w/ a second pass thru

Random Walk Ex

$p_i^t = P(i \text{ friends at time } t)$



Show for any stationary dist,

$\pi_i = \left(\frac{p}{1-p}\right)^i \pi_0$

$\pi_0 = (1-p)\pi_0 + (1-p)\pi_1 = (1-p)\pi_0 + (1-p)(p\pi_0 + (1-p)\pi_2)$

$\pi_1 = p \cdot \pi_0 + (1-p)\pi_2$

$\pi_i = p \cdot \pi_{i-1} + (1-p)\pi_{i+1}$

$(1-p)\,\pi_2 = \frac{p^2}{1-p}\pi_0$

So $\pi_i = \left(\frac{p}{1-p}\right)^i \pi_0$

What do we need for a valid π to exist?

$\sum_i \pi_i = 1 \rightarrow \sum_i \left(\frac{p}{1-p}\right)^i p_0 = \frac{p_0}{1 - \frac{p}{1-p}}$

we need this to converge. $p < 0.5$

Probability Things

1. Pr → exactly a times
   - → calc for one case
2. Show Pr at least a times with probability at most (fraction)
   - → Union bound w/ i tries, i > a $\forall i$
   - → Markov bound
   - Chernoff
3. $Pr[\text{indicator } \Sigma] \geq (\;) \leq$ some vague e term, or δ
   - → esp if you have expectation [Z], use Chernoff
   - → Pick a β and derive for desired probability
   - → If you want n in terms of δ, solve for n, δ using e term and some β

Chernoff — expectation related bound, w/ a sum of indicator vars
- tighter bound, restrictions on n

Markov — not exp bound, no limit on Z
- less tight bound, no restrict on Z